

Tests of Lorentz Violation in Atomic and Optical Physics*

Neil Russell

Physics Department, Northern Michigan University, Marquette, MI 49855, U.S.A.

Atomic physics can probe the Lorentz and CPT symmetries at the Planck level. Bounds on coefficients for Lorentz violation have been found using atomic clocks, masers, electromagnetic cavities, and Penning traps, among others, and in future it may be possible to place bounds using spectroscopy of antihydrogen atoms. The CPT '04 Meeting on CPT and Lorentz Symmetry was held in August 2004 in Bloomington, Indiana, USA, and covered Lorentz violation in all branches of physics. This report gives an overview of the recent advances in Lorentz-symmetry studies in atomic and optical physics.

1. Introduction

Lorentz symmetry is built into the conventional theories of particle physics and gravity. This situation is legitimate, given the lack of evidence for Lorentz violation. However, it is possible that nature is not exactly Lorentz symmetric, with violations occurring at a scale too small for past experiments to resolve. On dimensional grounds, effects can be expected to involve the Planck scale, where gravitational forces are comparable to the electromagnetic and nuclear forces. In the last 15 years, experimental sensitivities have improved rapidly, and it has become increasingly apparent that a variety of experiments can access Planck-scale effects. In atomic and optical physics, these include experiments with atomic clocks, masers, optical and microwave resonators, precision spectroscopy, and particle traps. Thus, Lorentz symmetry or violation is an experimental question that needs to be clarified. This question has received renewed and vigorous attention stimulated by the introduction of a framework for quantifying all possible Lorentz violations called the Standard-Model Extension, or SME [1, 2].

The SME is the usual Standard-Model lagrangian augmented with all possible Lorentz-violating terms constructed from Standard-Model fields that are invariant under Lorentz transformations of the observer's inertial frame. It allows Lorentz violation in all areas of physics from the large-scale gravitational sector to the small-scale quantum sector. The theoretical motivation for considering Lorentz violation is the possibility that it may occur in a unified theory of quantum gravity. At the fundamental level, one approach is through string theory, in which Lorentz violation could for example occur spontaneously [3].

Even in its minimal form, the SME contains several hundred coefficients for Lorentz violation. They carry spacetime indices, and transform as tensors under observer coordinate transformations. However, they are

fixed entities in spacetime, and so cannot be controlled in any way. Distinct SME coefficients exist for each particle type, and experiments are sensitive to differing combinations of coefficients. Since laboratories are not inertial, but rotate with the Earth, one type of Lorentz-violation signal involves sidereal variations in experimental observables. The many coefficients make Lorentz violation possible in a myriad of different ways. Although any given experiment can only examine a small part of the coefficient space, the possibility exists that an experiment from any area could reveal Lorentz violation. Dozens of experiments have already probed parts of the coefficient space, and efforts are continuing to improve precisions.

All the SME coefficients quantify Lorentz violation and some also quantify CPT violation. The CPT symmetry, associated with the combined reversal of charge, inversion of parity, and reversal of time, is closely related to Lorentz symmetry by the CPT theorem [4].

The Third Meeting on CPT and Lorentz Symmetry was held in Bloomington, Indiana, in August 2004 [5], and attracted participants from all parts of the globe. The conference encompassed experiment and theory of Lorentz violation from all sectors of physics, including ones involving electromagnetic cavities [6, 7, 8, 9, 10, 11, 12, 13], atomic physics [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29], mesons [30], muons [31], neutrinos [32], and the Higgs [33]. This CAMOP report focuses on the topics specific to atomic and optical physics. In particular, efforts to understand and measure coefficients for Lorentz violation in the photon and fermion sectors will be discussed.

2. Optical and Microwave Cavities

On the theoretical front, the prospect of Lorentz violation in the photon sector has received much attention [6], and on the experimental side, various searches for Lorentz violation with electromagnetic cavities have been performed or are being refined for future experiments. Typical cavities produce highly stable resonant frequencies in the optical or microwave regimes. Stability is monitored by comparison with a suitable second resonator, often another cavity. Signals of Lorentz violation include variations in the output frequency correlated

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with the orientation or direction of motion.

The set of possible Lorentz-violating signals in the electromagnetic sector is governed by coefficients $(k_F)_{\mu\alpha\beta\gamma}$ for CPT symmetry, and coefficients $(k_{AF})^\kappa$ for CPT violation [7]. The latter set is bounded at exceptionally high levels and will be assumed to be zero [8]. This leads to modified source-free inhomogeneous Maxwell equations:

$$\partial_\alpha F_\mu{}^\alpha + (k_F)_{\mu\alpha\beta\gamma} \partial^\alpha F^{\beta\gamma} = 0, \quad (1)$$

and unchanged homogeneous Maxwell equations:

$$\partial_\mu \tilde{F}^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\kappa\lambda} \partial_\mu F_{\kappa\lambda} = 0. \quad (2)$$

Solving these equations for $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ with the appropriate boundary conditions for a given cavity experiment gives the detailed form of the output frequency dependence on the $(k_F)_{\mu\alpha\beta\gamma}$ coefficients.

Due to the symmetries and other properties of $(k_F)_{\mu\alpha\beta\gamma}$, there are 19 independent components. Ten have been bounded by astrophysical data at impressive levels [9], and the remaining 9 are the focus of cavity experiments searching for Lorentz violation. At present, bounds have been achieved on 7 of these and sensitivities are steadily improving.

The analysis is aided by defining particular linear combinations of these components: five in the traceless symmetric matrix $\tilde{\kappa}_{e-}$, three in the antisymmetric matrix $\tilde{\kappa}_{o+}$, and one in the component $\tilde{\kappa}_{tr}$. These matrices $\tilde{\kappa}_{e-}$ and $\tilde{\kappa}_{o+}$ have spatial indices chosen in any suitable inertial reference frame. The coefficients for Lorentz violation in the SME represent geometrical objects fixed in spacetime, so the frequency output of a cavity oscillator would undergo cyclic variations as the laboratory rotates with the motion of the Earth relative to the fixed distant stars. In a typical experiment, the beat frequency of two cylindrical oscillators mounted at 90 degrees to each other would have the form

$$\begin{aligned} \frac{\nu_{beat}}{\nu} = & \mathcal{A}_{\oplus s} \sin \omega_\oplus T_\oplus + \mathcal{A}_{\oplus c} \cos \omega_\oplus T_\oplus \\ & + \mathcal{B}_{\oplus s} \sin 2\omega_\oplus T_\oplus + \mathcal{B}_{\oplus c} \cos 2\omega_\oplus T_\oplus + \mathcal{C}_\oplus. \end{aligned} \quad (3)$$

The components $\tilde{\kappa}_{e-}$, $\tilde{\kappa}_{o+}$, and $\tilde{\kappa}_{tr}$ appear together with geometrical factors and an annual time variation in the coefficients $\mathcal{A}_{\oplus s}$, $\mathcal{A}_{\oplus c}$, $\mathcal{B}_{\oplus s}$, $\mathcal{B}_{\oplus c}$, and \mathcal{C}_\oplus . An important signal of Lorentz violation is the time dependence at one or two times the Earth's sidereal frequency ω_\oplus . To allow comparison of results, a standard inertial reference frame is used, involving a Sun-centered coordinate system (X, Y, Z) with time T_\oplus based on an equinox in the year 2000.

Initial bounds of about 10^{-13} on components of $\tilde{\kappa}_{e-}$ and about 10^{-9} on components of $\tilde{\kappa}_{o+}$ were published by a Stanford-based group in 2003 [10]. These results were achieved with a pair of cylindrical superconducting cavity-stabilized oscillators operating in the TM_{010} mode with one east-west axis and one vertical axis. These limits have since been improved by about two orders of magnitude by one group based at German institutions

in Berlin, Düsseldorf, and Konstanz [11], and by another group associated with the Paris Observatory and the University of Western Australia [12].

Since one of the signals for Lorentz violation is a time dependence due to the rotation of the apparatus, sensitivity can be improved with a rotating turntable leading to a reduced period of oscillation and other advantages. All three groups mentioned above are in various stages of development in this direction. The Paris-based experiment has compared the output of a cryogenic sapphire microwave oscillator with a hydrogen maser, both running for several years. An improved experiment is under way at Western Australia, involving a rotating turntable with two sapphire cylinders within superconducting niobium cavities horizontally mounted with perpendicular axes. The German group has analyzed more than a year of output from two orthogonally mounted cryogenic sapphire resonators running in the optical regime. A refined experiment includes a precision turntable and better cryogenics to improve the sensitivity.

Electromagnetostatics has also been studied [13] for Lorentz-violation signals. Interesting effects include a small Lorentz-violating magnetic field for a stationary point charge, and a nonzero scalar potential within a conducting shell containing a magnetostatic source. Experiments searching for these effects could complement those done with cavities to study Lorentz violation in the photon sector.

3. Testing Lorentz symmetry with fermions

In the fermion sector, sensitive tests of Lorentz symmetry are possible with precision spectroscopy using masers, atomic clocks, particle traps, and possibly antihydrogen. In these systems, the fixed Lorentz-violating background is quantified for electrons by the coefficients a_μ^e , b_μ^e , $H_{\mu\nu}^e$, $c_{\mu\nu}^e$, and $d_{\mu\nu}^e$, where the e is for the couplings to electrons and would be replaced with p or n for protons or neutrons. The resulting modified Dirac equation with spinor ψ for an electron of mass m_e and charge $-q$ is:

$$\begin{aligned} (i\gamma^\mu D_\mu - m_e - a_\mu^e \gamma^\mu - b_\mu^e \gamma_5 \gamma^\mu \\ - \frac{1}{2} H_{\mu\nu}^e \sigma^{\mu\nu} + i c_{\mu\nu}^e \gamma^\mu D^\nu + i d_{\mu\nu}^e \gamma_5 \gamma^\mu D^\nu) \psi = 0. \end{aligned} \quad (4)$$

In a system such as hydrogen or antihydrogen, the Coulomb potential is contained in the vector potential A_μ and appears in the usual manner, via $iD_\mu \equiv i\partial_\mu - qA_\mu$. All the coefficients parameterize Lorentz violation, and a_μ^e and b_μ^e also parameterize CPT violation. The shifts in the energy levels can be calculated at leading order in perturbation theory [14]. For example, the shift in the hyperfine c to d transition of hydrogen in a 0.65-Tesla field is

$$\delta\nu_{c \rightarrow d}^H \approx -\frac{1}{\pi} (b_3^p - d_{30}^p m_p - H_{12}^p) \equiv -\frac{1}{\pi} \tilde{b}_3^p, \quad (5)$$

where the superscript indicates that the sensitivity is to proton coefficients. In this system, b_3^p is the component of

b_μ^p along the quantization axis defined by the magnetic-field direction; similarly, the 1 and 2 subscripts refer to the other two orthogonal directions in the laboratory reference frame.

Since the laboratory is rotating with the motion of the Earth, the appearance of variations in the above hyperfine frequency with the sidereal period of the Earth would indicate Lorentz violation. Experimental bounds on some of the components b_X^p , b_Y^p , b_Z^p , and b_T^p in the standard inertial reference frame can be attained by fitting to the appropriate time dependence. Sensitivity may be improved with the use of a rotating turntable. Sensitivity is also better for transitions with the smallest possible line width, other factors being equal. This would indicate that the 1S-2S transition in hydrogen is a candidate. However, calculations show that this transition is suppressed by a factor of the square of the fine-structure constant. Thus, the selection of optimal transitions involves knowledge of calculated suppressions and of attainable frequency resolutions.

Recent experiments [15] at the Harvard-Smithsonian Center for Astrophysics have used a hydrogen maser to place bounds on b_X^p and b_Y^p , components in the two equatorial directions, of 2×10^{-27} GeV. The b to d transition was used to search for sidereal variations, and the result is the sharpest proton-coefficient constraint to date.

When antihydrogen spectroscopy becomes available, another type of Lorentz test will be possible using the instantaneous comparison of the hydrogen spectrum with that of antihydrogen. In equation (5), the sign of the coefficient for CPT violation b_3^p is reversed for antihydrogen. So, a comparison between corresponding transitions for the two atoms will isolate the CPT-violating term only. This clean test cannot be achieved with searches for sidereal variations, which bound combinations involving also coefficients of Lorentz-violating, CPT-preserving terms. Of the three antihydrogen collaborations at CERN, the ASACUSA group plans to use an antihydrogen beam to measure the ground-state hyperfine splitting [16]. The ATHENA [17] and ATRAP [18] groups have made progress towards spectroscopy with trapped antiatoms.

Other experiments that compare matter and antimatter to measure cleanly coefficients for CPT violation include ones with trapped fermions in Penning traps [19]. The group at the University of Washington in Seattle placed bounds on SME coefficients based on sidereal and instantaneous-comparison measurements [20]. Other Penning-trap tests have been done and are under development at Harvard University [21]. Compton scattering may shed further light on the behavior of electrons in the Lorentz-violating background [22].

4. Clock-comparison experiments

The high stability of atomic clocks is well suited for performing searches for the sidereal effects of Lorentz violation. Analysis of cesium and other atoms common in

atomic clocks leads to a number of challenges not present for simpler systems such as hydrogen and antihydrogen because the analysis requires the use of nuclear models. However, an analysis of all possible Lorentz-violation signals on atomic clocks has been done [23]. This work includes some bounds based on existing experiments done in other contexts. For each particle species, there are five possible coefficient combinations that these experiments can probe, one of them being the laboratory-frame component \tilde{b}_3^p in equation (5). This component is one of the four components of \tilde{b}_μ^p , where μ refers the laboratory-frame coordinates. As with all the other experiments mentioned above, these components have to be related to the inertial-reference-frame components b_X^p , b_Y^p , b_Z^p , and b_T^p through a transformation that involves the rotation and speed of the Earth, the laboratory latitude, and other geometrical information. Thus, the parameter space for Lorentz violation with clock comparison tests is extensive. Fortunately, experiments are sensitive to different regions of this space, and rapid progress is being made in probing a large part of it.

Early bounds on SME coefficients in the fermion sector were obtained using a comparison between Cs and Hg magnetometers [24] by a group based at Amherst College.

An important aspect of these experiments is their exceptional frequency stability. A group at Princeton [25] is developing a $^3\text{He-K}$ comagnetometer designed to reduce spin-exchange line broadening, which should be extremely competitive.

A group at the Harvard-Smithsonian Center for Astrophysics has developed a colocated ^{129}Xe and ^3He Zeeman maser system. The one species is used as a magnetometer to stabilize the 1.5-Gauss magnetic field while the other species within the same bulb searches for sidereal variations due to Lorentz violation. The resulting bound on the equatorial components \tilde{b}_X^q and \tilde{b}_Y^q is at the level of 10^{-31}GeV [26]. Recently, this group has placed the first limits on boost effects in the neutron sector of the SME [27]. This type of signal indicates Lorentz violation under boost transformations, and is suppressed by the ratio $\beta_\oplus = v_\oplus/c = 9.9 \times 10^{-5}$ of the laboratory speed in the standard Sun frame relative to that of light.

The motion of the laboratory apparatus relative to the standard Sun-based reference frame is an important part of almost all Lorentz tests. It is an advantage to have rotation rates greater than the sidereal rotation rate of the Earth, and it is also beneficial to have large velocity changes relative to the Sun frame. Space platforms carrying atomic clocks or other oscillators are therefore interesting candidates for performing Lorentz tests. An analysis of such tests based on a satellite orbiting the Earth has been completed [28]. The International Space Station is expected to house various oscillators in the future, making such tests a possibility. The microgravity environment should make it possible to improve on fountain clocks that are limited by the gravitational field of the Earth. Other advantages include improved access to SME coefficients in the spatial components perpendicular

to the equatorial plane, and higher rotation rates. Free-flying missions may offer additional advantages by optimizing speeds, rotation rates, flight orientation modes, trajectory geometries, and other features.

5. Discussion

The SME Lorentz-violation framework encompasses all areas of physics. This breadth makes it challenging for each subdiscipline to absorb the progress being made in another, even though they are not independent. In fact, there are many cross-connections between Lorentz violation from the different sectors of the SME. For example, the photons oscillating in a crystal cavity are affected

also by deformations in the oscillator shape due to the fermions forming the crystal [29]. CPT '04 provided a valuable opportunity for interaction between experimentalists and theorists from diverse sectors including particle physics, atomic physics, gravity, astrophysics, and cosmology. Testing for Planck-scale effects is a major undertaking, and no experiment to date has found evidence of Lorentz violation. However, in the last 15 years, sensitivities of atomic and optical experiments have improved by many orders of magnitude. Any experiment that conclusively finds Lorentz violation will replace the 100-year era of Lorentz symmetry with a new era in which the minuscule violations will point towards the fundamental theory of quantum gravity.

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